NONSTATIONARY HEAT TRANSFER TO THE WALLS OF A CYLINDRICAL CHANNEL FILLED WITH LIQUID

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The results of an experimental investigation into the nonstationary (transient) heat transfer from an external medium to the walls of a dead-end cylindrical channel filled with liquid and closed by a copper bottom are presented. The experimental apparatus and method of conducting the experiments are described.

In this paper we shall be considering heat transfer from an external medium to the walls of vertical and inclined cylindrical channels in an infinite slab. The channel is filled with liquid, thermally insulated on one side and closed with a metal bottom on the other. We shall consider the case of a sudden change in the temperature of the external medium, for which heat transfer is mainly executed by natural convection of the liquid in the channel.

An analysis of the differential equations and uniqueness conditions describing heat transfer to the surface of the channel by generalized-variable methods yields the following laws for the Biot criterion and the dimensionless temperature of the liquid in the channel:

$$\mathrm{Bi} = A\,(\mathrm{Gr}\,\mathrm{Pr}^2)_0^n\,\mathrm{Fo}^m\,\mathrm{Bi}_{\mathrm{ex}}^p\left(\frac{T_0}{T_{\mathrm{ex}}-T_0}\right)^r, \quad (1)$$

$$\vartheta = B\,(\mathrm{Gr}\,\mathrm{Pr}^2)_0^x\,\mathrm{Fo}^y\mathrm{Bi}_{\mathrm{ex}}^z \quad (2)$$

In determining the form of these relationships, it was assumed that there was a functional relationship of the power type between the criterial quantities, and also that the simplexes a_b/a_{0l} and a_s/a_{0l} varied over a narrow range and had no effect on the values of Bi and ϑ.

The experiments were designed to evaluate the coefficients of proportionality and the power indices in (1) and (2). Three models were accordingly prepared in the form of hollow thin-walled cylinders. The geometrical dimensions and materials of the models are indicated in Table 1.

A copper bottom was attached to the lower part of model No. 1, and the channel was closed with a stopper at the top. A drain system was also placed at the top in order to compensate for the thermal expansion of the liquid on heating. Thirty six Chromel–Copel thermocouples were placed in four cross sections spaced over the height of the model. Models 2 and 3 had a similar construction.

The arrangement of the experimental apparatus is shown in Fig. 1. In order to establish an initial temperature of T_0 = 283-373°K the model was immersed in the thermostat 5a; in order to establish a temperature of T_0 = 273°K it was immersed in the thermostat 5b. The boundary conditions were established

TABLE 1. Dimensions and Materials of the Models

No. of model	External diameter, m	Diameter of channel, m	Length of model, m	Material
1	0.11	0,03	0,40	Textolite
2	0,08	0,02	0,40	organic glass 2-5
3	0,37	0,08	1,25	Textolite

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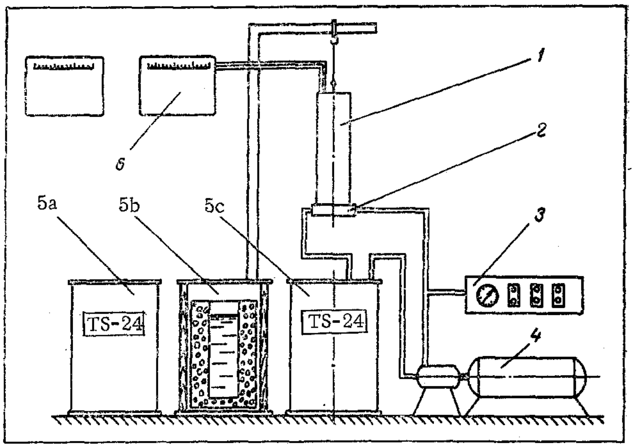

Fig. 1. Arrangement of the experimental apparatus: 1) model; 2) heat exchanger; 3) control desk; 4) transfer pump; 5a, b, c) thermostats; 6) measuring device.

by means of a special oil system comprising a thermostat 5c, a pump 4 with an oil-pressure regulator, and a heat exchanger 2. The thermostat 5c ensured a constant temperature of the oil T_{ex}. Using the pressure regulator, a specified velocity was imparted to the oil in the heat exchanger, and any desired heat-transfer coefficient relative to the bottom of the model was thus established for constant temperatures T_{ex} and T_0. In order to eliminate heat transfer along the sides, the model was thermally insulated with a thick felt casing. The channels of the model were filled with liquid (kerosene T-1, distilled water, or transformer oil).

In order to carry out the experiments on model No. 3 a special thermostat was designed and made.

By studying the results of the experiments, we determined the laws governing the changes taking place in the temperatures of the liquid and the surface of the channel with time, and from the resultant data we calculated the thermal flux and heat-transfer coefficient. To this end we first solved the problem of the temperature field of a hollow, infinite cylinder with time-varying boundary conditions of the first kind, and then determined the derivative of the temperature with respect to the radius.

The initial thermal fluxes to the bottom of the model were measured by means of copper calorimeters [1].

The first series of experiments was aimed at determining the power index of the Biot criterion in Eq. (1). For this purpose we specified a new value of α_{ex} in each experiment, while the remaining parameters determining the heat-transfer process remained constant. On the basis of the experimental results we constructed the $Bi = f(Fo, Bi_{ex})$ relationship. Analysis of the experimental curves showed that the $Bi = f(Fo)$ relation was satisfactorily described by the equation

$$Bi = C\,Fo\,(1 - 0.385\,Fo). \tag{3}$$

Allowing for (3), Eq. (1) may be written as follows:

$$Bi = D\,(Gr\,Pr^2)_0^n\,Bi_{ex}^p\left(\frac{T_0}{T_{ex} - T_0}\right)^r Fo\,(1 - 0.385\,Fo). \tag{4}$$

The relationship thus obtained enables us to analyze the results of the first series of experiments in the form

$$\lg Bi - \lg Fo\,(1 - 0.385\,Fo) = F + p \lg Bi_{ex}. \tag{5}$$

We found that Eq. (5) was almost linear, with $p = -0.4$.

In the next series of experiments, the initial temperature of the model and the temperature of the external medium were varied so that the ratio $T_0/(T_{ex} - T_0)$ remained almost the same from one experiment to another. Analysis of the experimental results in the form of the relation

$$\lg Bi - \lg Fo\,(1 - 0.385\,Fo) + 0.4 \lg Bi_{ex} = f\,(Gr\,Pr^2)_0$$

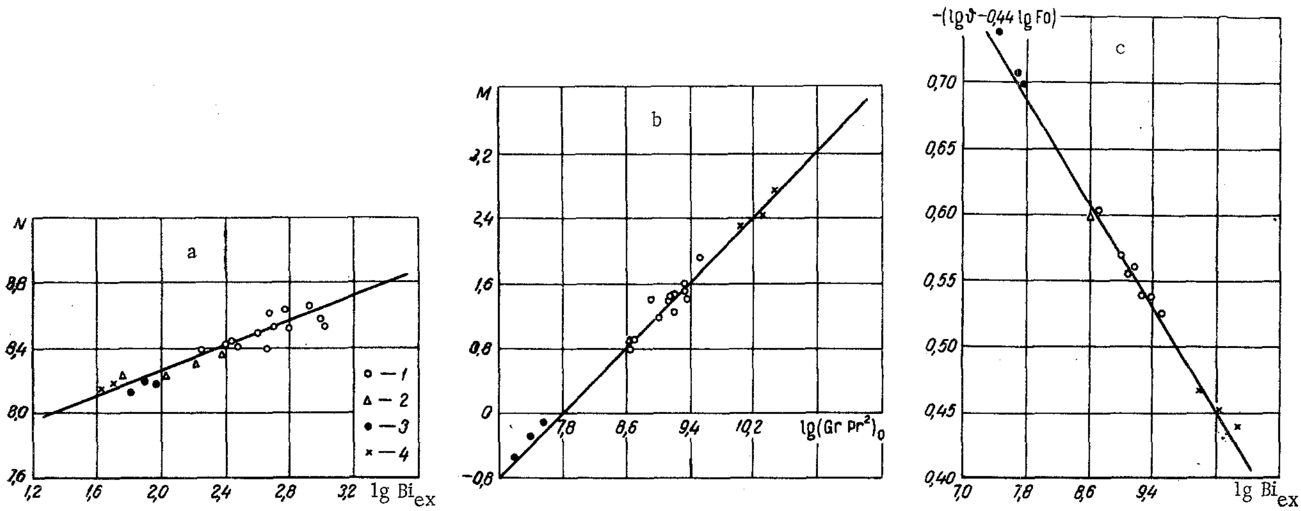

Fig. 2. Dependence of $N = -\{\log \mathrm{Bi} - \log[\mathrm{Fo}(1 - 0.385\ \mathrm{Fo})] + 0.45 \log(\Delta T/T_0) - \log(\mathrm{GrPr}^2)_0\}$ on $\log \mathrm{Bi}_{ex}$ (a); $M = \log \mathrm{Bi} + 0.45 \log(\Delta T/T_0) + 0.4 \log \mathrm{Bi}_{ex} - \log\ \mathrm{Fo} \cdot (1 - 0.385\ \mathrm{Fo})$ on $\log(\mathrm{GrPr}^2)_0$ (b) and log $- 0.44 \log \mathrm{Fo}$ on $\log(\mathrm{GrPr}^2)_0$ (c) (the curves in a and b were calculated by Eq. (6) and those in c by Eq. (7)): 1) d = 0.03 mm, working substance kerosene T-1; 2) 0.02 m, kerosene; 3) 0.02 m, water; 4) 0.08 m, transformer oil.

enabled us to determine the power index of the criterion $(\mathrm{GrPr}^2)_0$ n = 1. Thus Eq. (1) transforms into the form

$$\mathrm{Bi} = D\,(\mathrm{Gr\,Pr}^2)_0\,\mathrm{Bi}_{ex}^{-0.4}\,\mathrm{Fo}\,(1 - 0.385\,\mathrm{Fo})\left(\frac{T_0}{T_{ex} - T_0}\right)^r,$$

from which it follows that the results of experiments with any arbitrary combination of the parameters determining heat transfer may be used in order to find the constants D and r.

The proportionality factors and power indices in Eq. (2) were determined in an analogous manner.

Analysis of the experimental data enabled us to establish the form of the functional relationship for the Biot criterion and the dimensionless temperature of the liquid averaged over the height of the channel:

$$\mathrm{Bi} = 1{,}547 \cdot 10^{-8}\,\frac{\mathrm{Fo}\,(1 - 0.385\,\mathrm{Fo})\,(\mathrm{Gr\,Pr}^2)_0}{\mathrm{Bi}_{ex}^{0.4}\left(\dfrac{T_{ex} - T_0}{T_0}\right)^{0.45}}, \tag{6}$$

$$\vartheta = 0.034\,\mathrm{Fo}^{0.44}\,(\mathrm{Gr\,Pr}^2)_0^{0.1}. \tag{7}$$

It follows from Fig. 2 that the calculated relationships satisfactorily describe the experimental data, thus confirming the validity of Eqs. (6) and (7) for vertical channels having the parameter ranges:

$$\mathrm{Fo} = 0 - 1.6;\quad \mathrm{Bi}_{ex} = 50 - 1000;\quad (\mathrm{Gr\,Pr}^2)_0 = 10^7 - 10^{11};$$

$$\frac{T_{ex} - T_0}{T_0} = 0.025 - 0.366;\quad \frac{l}{d} = 13 - 20;\quad \frac{\delta}{d} = 0.03 - 0.13.$$

The experiments aimed at determining the boundary conditions on the side of the channel surface were accompanied by visual observations of the hydrodynamic characteristics of the convection currents, using transparent models. As models we used glass tubes of various diameters filled with liquid. The tubes were closed at the top and bottom with rubber stoppers, and a heater was mounted in the lower one of these.

Comparison between the external thermal fluxes taking place in the experiments with models Nos. 1 and 2 and the specific powers of the heaters characterizing the transition from one mode of operation to another in the transparent models showed that in all the experiments the mode of liquid flow in the channel was turbulent. This was confirmed by the absence of transverse temperature differences in any section of the channel: the temperatures measured at diametrically opposite points of the channel were almost exactly the same.

When the transparent model deviated from the vertical, the turbulent mode was replaced by a transitional and then by a laminar mode. When the experiment was carried out with models Nos. 1 and 2, set at a certain angle to the horizontal, we observed a transverse temperature difference; the difference equalled 2-5°K, depending on the external thermal flux of the section under consideration and the inclination of the model. The existence of the transverse temperature difference indicated a laminar type of flow of the liquid in the channel.

The transition from the more intensive to the less intensive mode of flow leads to a reduction in the thermal flux passing to the wall of the channel. From the results of the experiments we may recommend the following approximate relationship for the thermal flux passing to the wall of a channel with its axis inclined at an angle of γ $(0 \ll \gamma \ll 70°)$ to the vertical:

$$q_\gamma = 0.6q\left(1 + \frac{2}{3}\cos\gamma\right), \tag{8}$$

where q is the thermal flux to the wall of the channel in the vertical position.

NOTATION

T	is the temperature;
τ	is the time;
d, d	is the diameter of the channel;
δ	is the thickness of the metal bottom;
g	is the gravitational acceleration;
α, a, λ, ν, β	are the heat-transfer coefficient, thermal diffusivity, thermal conductivity, kinematic viscosity, and volume expansion coefficient;
A, B, C, D, F, n, m, p, r, x, y, z	are constants in the criterial relationships;
l	refers to the parameters of the liquid inside the channel;
s	refers to the parameters of the slab;
b	refers to the parameters of the bottom;
ex	refers to the parameters of the external medium;
0	refers to the initial value of the parameters.

$\vartheta = (T_l - T_0)/(T_{ex} - T_0)$;
$Gr_0 = g\beta_{0l}\Delta T d^3/\nu_{0l}^2$;
$Pr_0 = \nu_{0l}/a_{0l}$;
$Fo = a_{0l}\tau/(d/2)^2$;
$Bi_{ex} = \alpha_{ex}(d/2)/\lambda_{0l}$;
$Bi = \alpha_l(d/2)/\lambda_s$;
$\Delta T = T_{ex} - T_0$.

LITERATURE CITED

1. E. V. Kudryavtsev, K. N. Chakalev, and N. V. Shumakov, Nonstationary Heat Transfer [in Russian], Izd. Akad. Nauk SSSR (1961).